ELIZADE UNIVERSITY, ILARA-MOKIN,
ONDO STATE, NIGERIA

DEPARTMENT OF
MECHANICAL, AUTOMOTIVE AND PRODUCTION ENGINEERING

FIRST SEMESTER EXAMINATIONS

2017/2018 ACADEMIC SESSION

HOD'S SIGNATURE

COURSE: MEE 503 – Theory of Elasticity (3 Units)

CLASS: 500 Level Mechanical Engineering

TIME ALLOWED: 3 Hours

INSTRUCTIONS: Answer any **FOUR** questions

Date: March, 2018

Question 1

(a) With respect to the frame of reference oxyz, the following state of stress exist.

$$\tau_{ij} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Determine the principal stresses.

(b) For the state of stress shown in Figure 1b, determine

- The element orientation for the principal stresses
- The principal stresses
- The maximum shearing stress and the corresponding normal stress

(25 Marks)

Question 2

(a)

(i) What are the methods for reducing stress concentration?

(ii) The flat bar shown in Figure 2a is 10 mm thick and is pulled by a force P producing a total change in length of 0.2mm. Determine the maximum stress developed in the bar (E = 200 Mpa).

(b)The stresses acting on element A in the fuselage of an airplane are found to be 40 MPa tension in the horizontal direction and 160 MPa compression in the vertical direction (See Figure 2b). The shear stresses of magnitude 54 MPa act in the direction shown. Determine the stresses acting

on an element oriented at a clockwise angle of $52°$ from the horizontal. **(25 Marks)**

Question 3

(a) A brass cube 50 mm on each edge is compressed in two perpendicular directions by forces P = 175 kN (See Figure 3a). Calculate the change ΔV in the volume of the cube and the strain energy U stored in the cube, assuming E = 100 GPa and $\nu = 0.34$.

(b) A rectangular plate (see Figure 3b) is subjected to normal stresses

$$\sigma_{xx} = 90\ MPa,\ \sigma_{yy} = -20\ MPa$$

The plate has dimensions 400 x 800 x 20 mm and it is made of steel with E = 200 GPa and $\nu =$ 0.30.

- Determine the maximum in-plane shear strain in the plate
- Determine the change ΔV in the volume of the plate

(25 Marks)

Question 4

(a) Briefly describe the following experimental techniques, namely;

- Strain Gauges
- Photoelasticity
- Moire Method

(b)The strain on the surface of an experimental device made of pure aluminum (E = 70 GPa and $\nu = 0.35$) were measured by means of strain gauges. The gauges were oriented as shown in the Figure 4b and the measured strains were

$$\varepsilon_a = 1100x10^{-6}, \varepsilon_b = 1496x10^{-6}, \varepsilon_c = -39.44x10^{-6}$$

Calculate the stress σ_{xx} in the device.

(25 Marks)

Question 5

(a) Outline the steps involved in using the Finite Element Method for solving engineering problems.

(b) For the spring system shown in Figure 5b

$k_1 = 100\,N/mm,\ k_2 = 200\,N/mm,\ k_3 = 100\,N/mm,\ P = 500N,\ u_1 = u_4 = 0$

Find the following, namely;

- The global stiffness matrix
- The displacement of nodes 2 and 3
- The forces in the spring 2

(25 Marks)

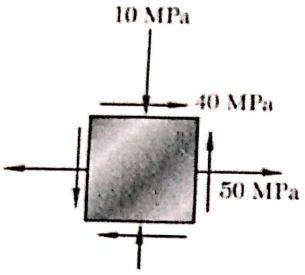

Figure 1b

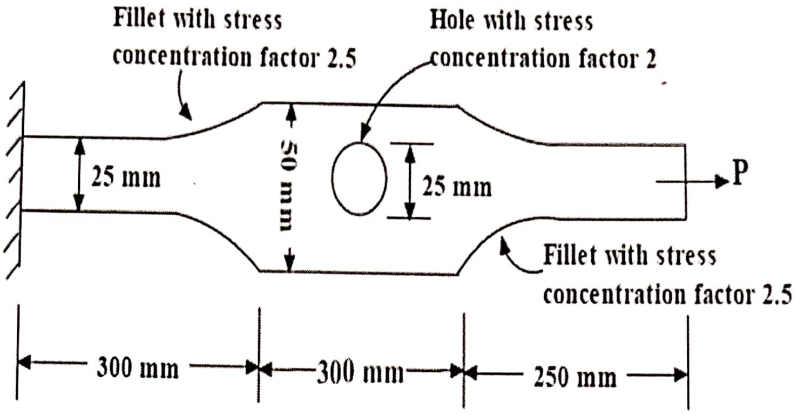

Figure 2a

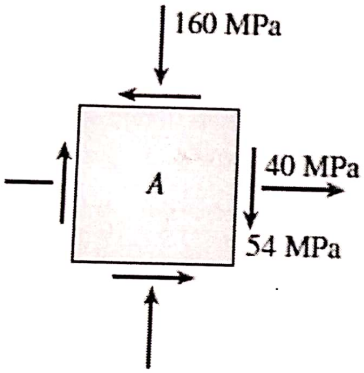

Figure 2b

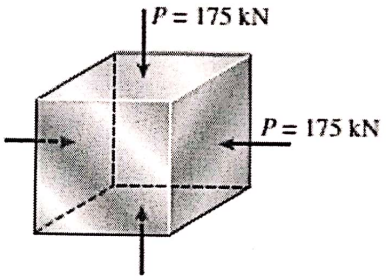

Figure 3a

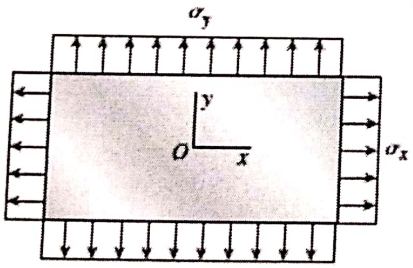

Figure 3b

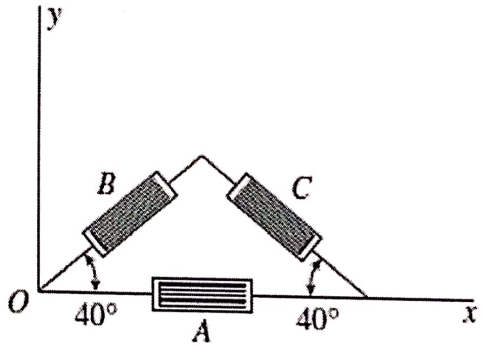

Figure 4b

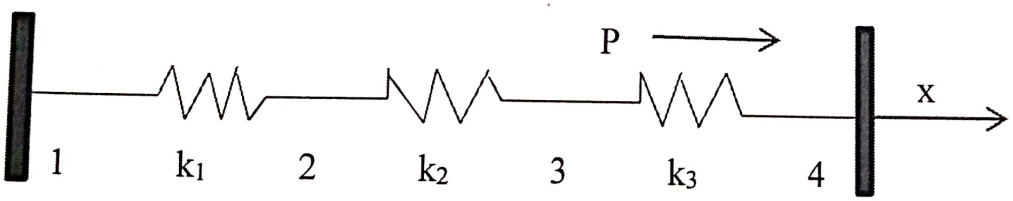

Figure 5b

SOME USEFUL FORMULAE MEE503

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\sigma_x = \frac{E}{(1-\nu)^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\Delta V = V_0\left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y)$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu(\varepsilon_x + \varepsilon_z)]$$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1$$